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Logarithmic Behavior as Metaphor

The public confusion of logarithmic growth with exponential growth first drew my attention at the 1966 Opening Convocation lecture to our college. In the course of this talk the speaker said, "As opposed to all other appetites which are stimulated by deprivation and satisfied by food, good education stimulates with plenty so that appetite for knowledge and understanding escalate logarithmically to insatiability." Afterwards, I gently tried to explain that exponential growth would more appropriately suggest the intended rapidity. Subsequently I've come across a possible explanation for the use of "logarithmically" to describe rapid increase: In bivariate data representation the adjustment of a variable so as to obtain a straight line relationship is often desirable, as Mosteller, Rourke and Thomas pointed out in their classic text [6]. Many data are "straightened out" by plotting the logarithm of one variable against the other variable. In particular, exponential growth when plotted on such "semi-log" paper shows up as a linear graph. (The increments on the vertical scale of semi-log paper are unequal in value: They are an exponential of the equally spaced values on a traditional y-axis.) In some disciplines, such as biology, "logarithmic growth" actually refers to what we mathematicians know as exponential growth because of this method of linearizing the data. That must underlie one of the most prominent such usages, nuclear scientist Ralph Lapp's *The Logarithmic Century*, in which he writes about the rapid growth of resource consumption, etc. [4] Interestingly, the dust jacket blurb uses, instead, the terminology "exponential growth". I would expect Lapp to have been aware of a choice of descriptions, and why he opted for "logarithmic" is beyond me. In any case, he helped keep alive an unfortunate terminology.

Since the occasion of the Opening Convocation I've run across other such examples, largely from sources with no apparent scientific culture to have misled them. Some of these are recounted below. Although many people still misuse the terminology, many display an awareness of the proper distinctions at issue. Perhaps with the help of those who teach about rates of change, computer algorithms, L'Hôpital's rule and the like, "logarithmic growth" and "exponential growth" will avoid the fate of "reticent" and "reluctant", which have become essentially synonymous.

Here are three examples of metaphorical growth from the New York Times, with the third suggesting an improved understanding on the part of this newspaper:

From a review of Harlow Shapley's autobiography: "if the autobiographer opts for a ... method he believes will grant him immortality without industry, his risks rise logarithmically." [8]

From a story about corruption and drugs: " 'The drug situation is a horror story, increasing logarithmically.' " [9]

From a more recent story: "Street crime, fed by an explosion of drug abuse, has risen exponentially." [10]

These sorts of examples also come up on television, as shown in this quote from Tim Daggett's NBC Olympics gymnastics commentary, July 23, 1996: "We have seen Kerri Strug's confidence increase almost logarithmically this year." [5]

Not even my own colleagues are immune to the confusion, as the following quote from our college's alumni magazine captures. A member of the English Department, with a particular interest in linguistics, complimented a student: "I observed that the experience of doing a translation made her grow logarithmically." [1]

A fellow mathematician brought to my attention the following gaffe, by a nationally prominent pediatrician: "The stresses parents are under have increased logarithmically since I was raising my children in the '60s and '70s." [3]

Of course the Internet, a source of so much value yet also so much error and disinformation, offers a host of examples, especially self-referential ones. For instance:

From a report on the state of technology, "The Internet is growing logarithmically." [13]

From Hunter S. Thompson on ESPN, “the Meanness quotient of the U.S. image in the world is growing logarithmically with every passing day.” [14]

From an editor at O'Reilly & Associates, which specializes in books on Unix, the Internet, etc.: “the logarithmic growth of the Web.” [15]

From a sales manager for Sun Microsystems, “The Internet’s Logarithmic Growth... Websites Doubling Every 57 Days.” [16]

And from a magazine aimed at electrical contractors, “a tens-of-billions-of-dollars market that is growing logarithmically.” [17]

As John Allen Paulos has noted, the confusion also arises in non-technical commercial promotions: “One real estate development company advertised that an investment with it would grow logarithmically.” [11]

Finally, as a result of my discussing this topic when teaching calculus, a student spotted the following juicy example: "Following his death, Malcolm's influence expanded in dramatic, almost logarithmic, fashion." [12] Malcolm X was a courageous fellow and deserves better than this.

The above quotes evidence a widespread public confusion, extending even to those who should know better. However, it’s worth giving two examples of writers in the arts who make proper use of logarithm as metaphor. Rather than suggesting a rapidity of growth, each uses the logarithm to connote sheer complexity. In his 1947 review of a film noir classic, Bosley Crowther wrote, “the sum of deceitful complications that occur in ‘Out of the Past’ must be reckoned by logarithmic tables, so numerous and involved do they become.” [7] More cryptically, the celebrated poet John Ashbery’s most famous work, “Self-Portrait in a Convex Mirror” contains the description,

“New York

Where I am now, which is a logarithm

Of other cities.” [2]

It looks as though these two prominent writers have perfect pitch in this area! Unfortunately, commentators across a broad range of careers still mangle the subject. With

luck, the examples presented here will encourage CMJ readers to do their part to stamp out this form of illiteracy.

References

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5. Miscellaneous quotation [25] from the Mathematical Olympiad Summer Program quotations list of Michael Freiman, <http://mail.med.upenn.edu/~freiman/MOPQuote.htm>
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12. William L. Van Deburg, *New Day in Babylon: The Black Power Movement and American Culture, 1965-1975*, Univ. of Chicago Pr., 1992, p. 2.
13. <http://www.ndu.edu/sdcfp/FedEx%20Final%20Report.doc> (June 2000 report on Federal Express for the Secretary of Defense Corporate Fellows Program, 3rd heading on p. 24), and, independently, <http://www.hsmai.org/releaseDetail.cfm?id=130> , (trend number 8.) This latter source “was selected by the Institute of Industrial Engineers as one of the top 10 Industrial Engineers in the U.S.”
14. <http://espn.go.com/page2/s/thompson/020328.html> (11th paragraph)
15. <http://www.empowermentzone.com/cgiintro.txt>
16. <http://www.ifla.org/IV/ifla62/62-kenc.htm> (Last lines of Chapter II, “Sun and the Internet”, just before "Java".)
17. http://www.ecmag.com/mk_p6.asp (Fifth bulleted item.)